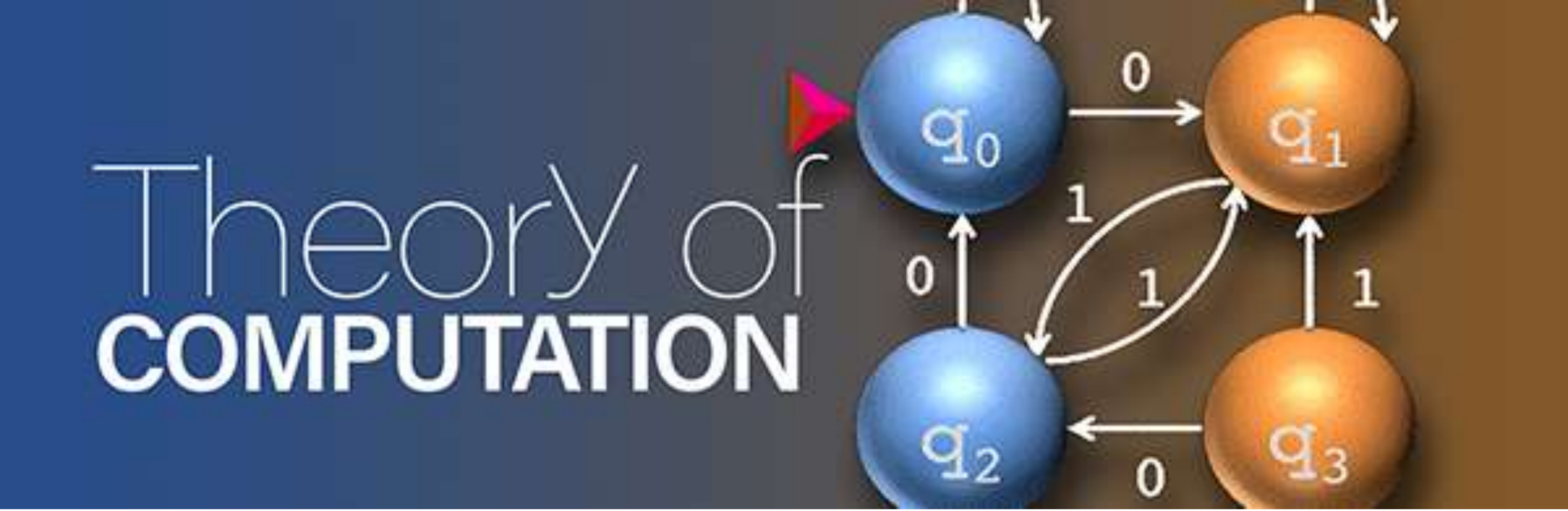

Lecture 8

Mathematical Preliminaries (4)

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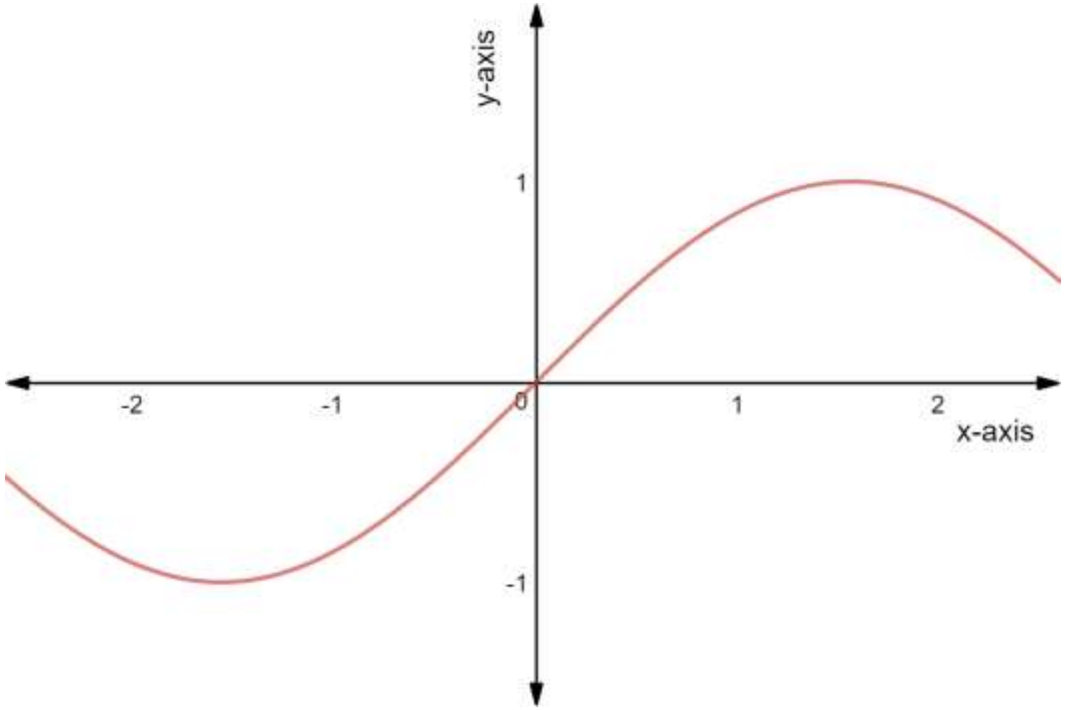
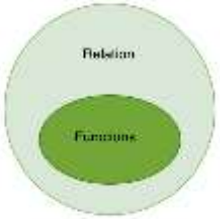


Function Definition

- **A function is a special kind of relation. It is a relation in which each domain value maps only to one range value.**
- It is denoted by ***f*:X⇒Y**
- What this means that it is a function from X to Y. It takes input from set X and gives the unique value from set Y as output. **"X" is called the domain of the function while "Y" is called the co-domain.**

What makes a relation to a function?

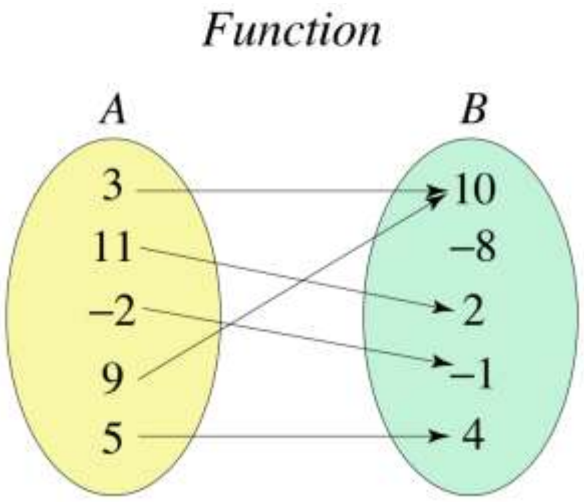
- **A relation in which an element is mapped to only range value is called a function**. To determine if a relation is a function, we just need to make sure that no element has two corresponding range values.

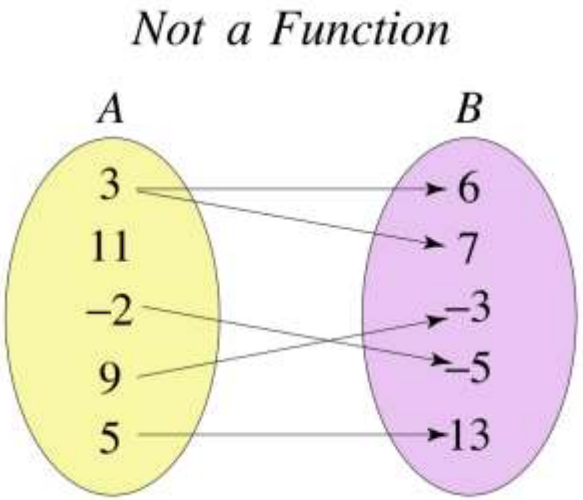


Function Definition

- A function, denoted by *f*, is a special type of binary relation. A function from set A to set B is a relation $f \subseteq A \times B$ that satisfies the following two properties:
  1. Each element a∈A is mapped to some element b∈B.
  2. Each element a∈A is mapped to exactly one element b∈B.
- If f is a function from set A to set B, we write f:A⟶B. The fact that a function *f* maps an element a∈A to an element b∈B is usually written as f(a)=b.

- In the example, the second relation (on the below) is NOT a function since both conditions are not met.





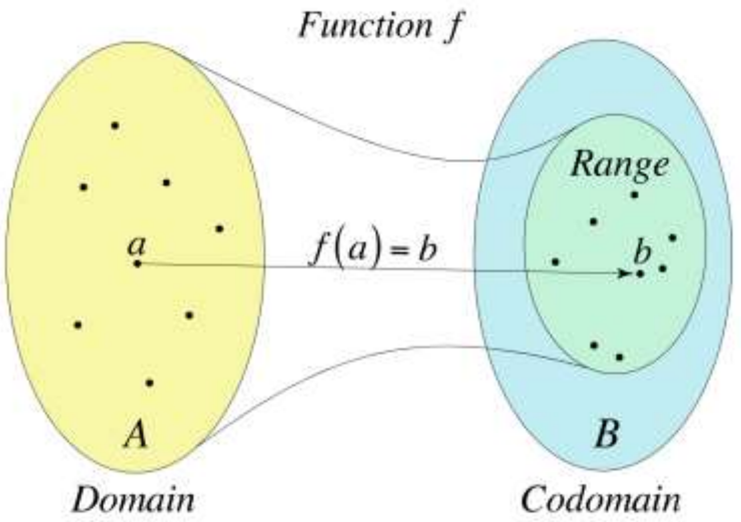
Function: Domain and Codomain

Domain, Codomain, Range, Image, Preimage:

- Consider a function f:A→B. The set A is called **the domain of the function f**, and **the set B is the codomain**. The domain and codomain of f are denoted, respectively, **dom(f)** and **codom(f)**.

- If f(a)=b, **the element b is the image of a under f**. Respectively, **the element a is the preimage of b under f**. The element a is also often called **the argument or input of the function f**, and the element b is called **the value of the function f** or its output.

- The set of all images of elements of A is briefly referred to as **the image of A**. It is also known as **the range of the function f**, although this term may have different meanings. The range of f is denoted **rng(f)**. It follows from the definition that **the range is a subset of the codomain**.

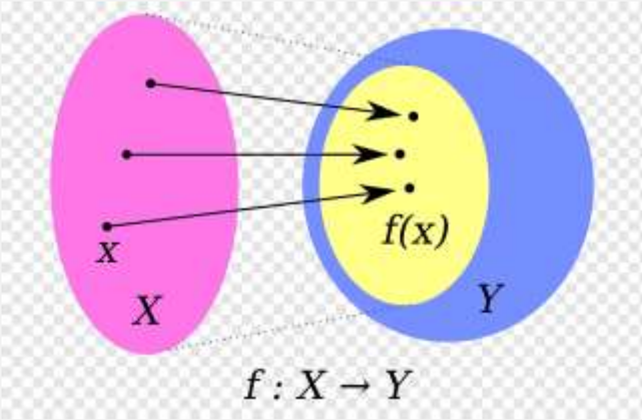


Representation of Functions

- **There are three different forms of representation of functions.**
- The functions need to be represented to showcase the domain values and the range values and the relationship between them.
- The functions can be represented with the help of:
  1. **Venn diagrams,**
  2. **Graphical formats, and**
  3. **Roster forms.**

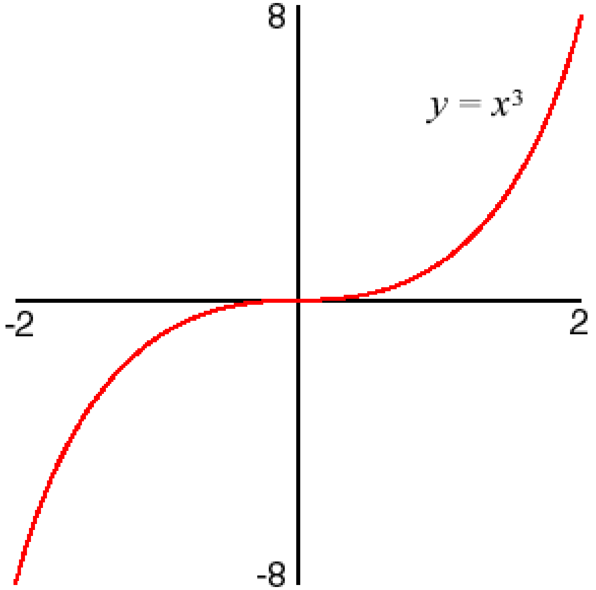
Representation of Functions

- **Venn Diagram:**
- The Venn diagrams are usually presented as two circles with arrows connecting the element in each of the circles.
- The domain is presented in one circle and the range values are presented in another circle.
- And the function defines the arrows, and how the arrows connect the different elements in the two circles.



Representation of Functions

- **Graphical Form:**
- Functions are easy to understand if they are represented in the graphical form with the help of the coordinate axes.
- **The domain of the function, the *x* value, is represented along the *x*-axis, and the range *y* or the f(*x*) value of the function is plotted with respect to the *y*-axis.**



Representation of Functions

- **Roster Form:**
- Roster notation of a set is a simple mathematical representation of the set in mathematical form.
- **The domain and range of the function are represented in flower brackets with the first element of a pair representing the domain and the second element representing the range.**

For Example:

- For a function of the form $f(x) = x^2$, the function is represented as {(1, 1), (2, 4), (3, 9), (4, 16)}.
- Here the first element is the domain or the *x* value and the second element is the range or the f(*x*) value of the function.

Types of Functions

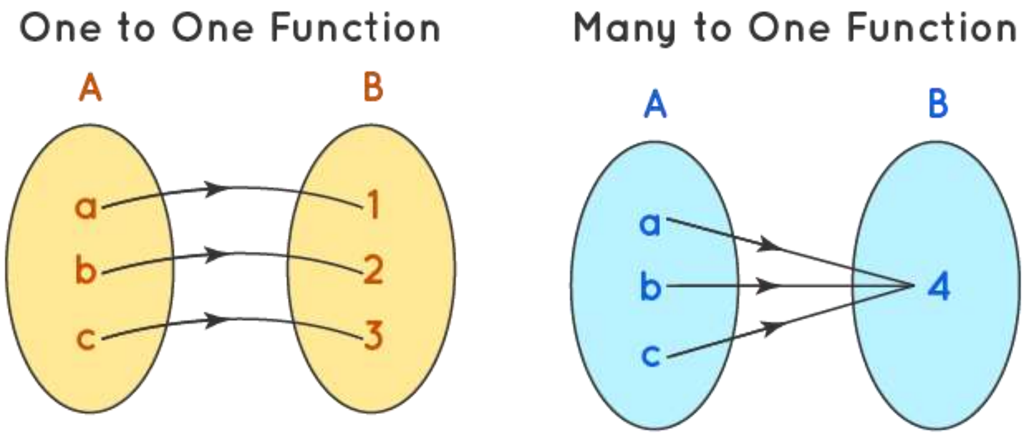
- The types of functions have been classified into the following four types:
    1. Based on the Set Elements
    2. Based on Equation
    3. Based on Range
    4. Based on Domain

- **Based on the Set Elements:**
    - Injective (One-to-One) Functions
    - Many-One Functions
    - Surjective (Onto) Functions
    - Into Functions
    - Bijective (One-to-One Onto) Functions
    - One-One Into Functions
    - Many-One Into Functions
    - Many-One Onto Functions
    - Constant Function

- **Based on Equation:**
    - Identity Function
    - Linear Function
    - Quadratic Function
    - Cubic Function
    - Polynomial Functions
- **Based on Range:**
    - Modulus Function
    - Rational Function
    - Signum Function
    - Even and Odd Functions
    - Periodic Functions
    - Greatest Integer Function
    - Inverse Function
    - Composite Functions
- **Based on Domain:**
    - Algebraic Functions
    - Trigonometric Functions
    - Logarithmic Functions

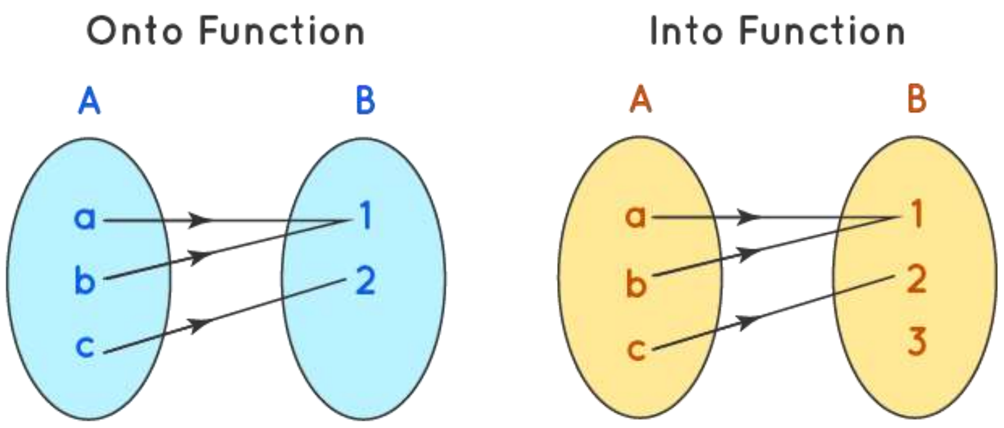
Types of Functions: Based on Set Elements

- **One-to-One Function:** A one-to-one function is defined by f: A → B such that every element of set A is connected to a distinct element in set B. Thus, in one-to-one function, one element of Domain Set is connected to one element of Co-Domain Set. The one-to-one function is also called an **injective function**.
- **Many to One Function:** A many to one function is defined by the function f: A → B, such that more than one element of the set A are connected to the same element in the set B. In a many to one function, more than one element has the same co-domain or image.



Types of Functions: Based on Set Elements

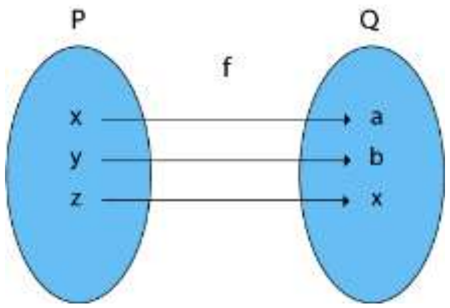
- **Onto Function:** In an onto function, every codomain element is related to the domain element. For a function defined by f: A → B, such that every element in set B has a pre-image in set A. The onto function is also called a **subjective function.**
- **Into Function:** The into function is exactly opposite in properties to an onto function. Here there are certain elements in the co-domain that do not have any pre-image. The elements in set B are excess and are not connected to any elements in set A.



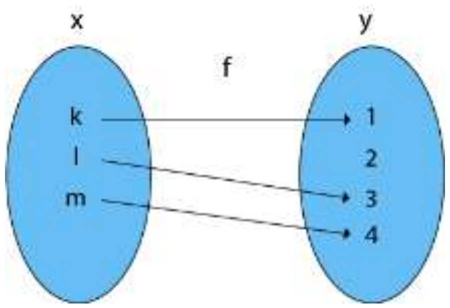
Types of Functions: Based on Set Elements

- **One-to-One Onto Function (Bijection):** A function that is both a one-to-one and onto function is called a bijective function. Here every element of the domain is connected to a distinct element in the codomain and every element of the codomain has a pre-image.
- **One-to-One Into Functions:** Let f: X → Y. The function f is called one-one into function if different elements of X have different unique images of Y.

One-to-One Onto



One-to-One Into



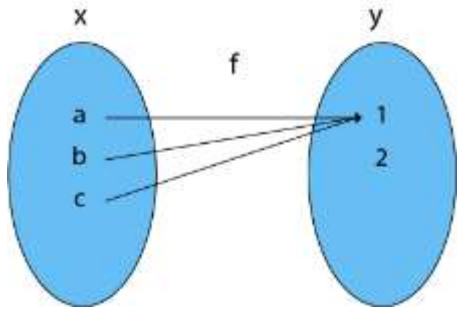
Types of Functions: Based on Set Elements

- **Many-to-One Into Functions:** Let f: X → Y. The function f is called the many-one function if and only if is both many one and into function.

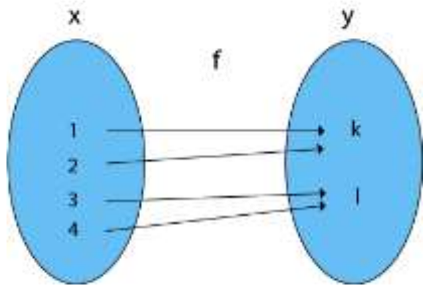
- **Many-to-One Onto Functions:** Let f: X → Y. The function f is called many-one onto function if and only if is both many one and onto.

- **Constant Function:** A constant function is an important form of a many to one function. In a constant function, all the domain elements have a single image.

Many-One Into



Many-One Onto



Types of Functions: Based on Equations

- The algebraic expressions are also functions and are based on the degree of the polynomial. The functions based on equations are classified into the following equations based on the degree of the variable 'x'.
  - The polynomial function of degree zero is called a Constant Function.
  - The polynomial function of degree one is called a Linear Function.
  - The polynomial function of degree two is called a Quadratic Function.
  - The polynomial function of degree three is a Cubic Function.
- **Identity Function:** The identity function has the same domain and range. The identity function equation is f(x) = x, or y = x.
- **Linear Function:** A polynomial function having the first-degree equation is a linear function. The general form of a linear function is f(x) = ax + b.
- **Quadratic Function:** A quadratic function has a second-degree quadratic equation and it has a graph in the form of a curve. The general form the the quadratic function is $f(x) = ax^2 + bx + c$, where $a \neq 0$ and a, b, c are constant & x is a variable.
- **Cubic Function:** A cubic function has an equation of degree three. The general form of a cubic function is $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$ and a, b, c, and d are real numbers & x is a variable.
- **Polynomial Function:** The general form of a polynomial function is $f(x) = a^n x^n + a^{n-1}x^{n-1} + a^{n-2}x^{n-2} +$ ..... ax + b. Here n is a nonnegative integer and x is a variable.

Types of Functions: Based on Range

- **Modulus Function:** The modulus function gives the absolute value of the function, irrespective of the sign of the input domain value. The modulus function is represented as f(x) = |x|. The input value of 'x' can be a positive or a negative expression.
- **Rational Function:** A function that is composed of two functions and expressed in the form of a fraction is a rational function. A rational fraction is of the form f(x)/g(x), and $g(x) \neq 0$.
- **Signum Function:** The signum function helps us to know the sign of the function and does not give the numeric value or any other values for the range. The range of the signum function is limited to {-1, 0, 1}. For the positive value of the domain, the signum function gives an answer of 1, for negative values the signum function gives an answer of -1, and for the 0 value of a domain, the image is 0. The signum function has wide application in software programming.
- **Even and Odd Function:** The even and odd functions are based on the relationship between the input and the output values of the function. If f(-x) = f(x), for all values of x, then the function is an even function, and if f(-x) = -f(x), for all values of x, then the function is an odd function. An example of even functions are x^2, Cosx, Secx, and an example of odd functions are x^3, Sinx, Tanx.

Types of Functions: Based on Range

- **Periodic Function:** The function is considered a periodic function if the same range appears for different domain values and in a sequential manner. The trigonometric functions can be considered periodic functions. For example, the function f(x) = Sinx, have a range equal to the range of [-1, 1] for the different domain values of $x = n\pi + (-1)^n x$.
- **Inverse function:** The inverse of a function f(x) is denoted by $f^{-1}(x)$. For inverse of a function the domain and range of the given function is reverted as the range and domain of the inverse function. The domain of Sinx is R and its range is [-1, 1], and for $Sin^{-1}x$ the domain is [-1, 1] and the range if R. The inverse of a function exists, if it is a bijective function. If a function $f(x) = x^2$, then the inverse of the function is $f^{-1}(x) = \sqrt{x}$.
- **Greatest Integer Function:** The greatest integer function is also known as the step function. The greatest integer function rounds up the number to the nearest integer less than or equal to the given number. The greatest integral function is denoted as $f(x) = \lfloor x \rfloor$. For a function taking values from [1, 2), the value of f(x) is 1.
- **Composite Function:** The composite functions are of the form of gof(x), fog(x), h(g(f(x))), and is made from the individual functions of f(x), g(x), h(x). The composite functions made of two functions have the range of one function forming the domain for another function. If f(x) = 2x + 3 and g(x) = x + 1 we have fog(x) = f(g(x)) = f(x + 1) = 2(x + 1) + 3 = 2x + 5.

Types of Functions: Based on Domain

- **Algebraic Function:** The algebraic function has a variable, coefficient, constant term, and various arithmetic operators such as addition, subtraction, multiplication, division. An algebraic function is generally of the form of f(x) = $a^n x^n + a^{n-1} x^{n-1} + a^{n-2} x^{n-2} +$ .... ax + c.
- The algebraic function is also termed as a linear function, quadratic function, cubic function, polynomial function, based on the degree of the algebraic equation.
- **Trigonometric Functions:** The trigonometric functions also have a domain and range similar to any other function. The six trigonometric functions are f(θ) = sinθ, f(θ) = cosθ, f(θ) = tanθ, f(θ) = secθ, f(θ) = cosecθ. Here the domain value θ is the angle and is in degrees or in radians. These trigonometric functions have been taken based on the ratio of the sides of a right-angle triangle, and are based on the Pythagoras theorem.
- **Logarithmic Functions:** Logarithmic functions have been derived from the exponential functions. The logarithmic functions are considered as the inverse of exponential functions. Logarithmic functions have a 'log' in the function and it has a base. The logarithmic function is of the form $y = \log_a X$. Here the domain value is the input value of 'x' and is calculated using the Napier logarithmic table. The same logarithmic function can be expressed as an exponential function as $x = a^y$.

THE END