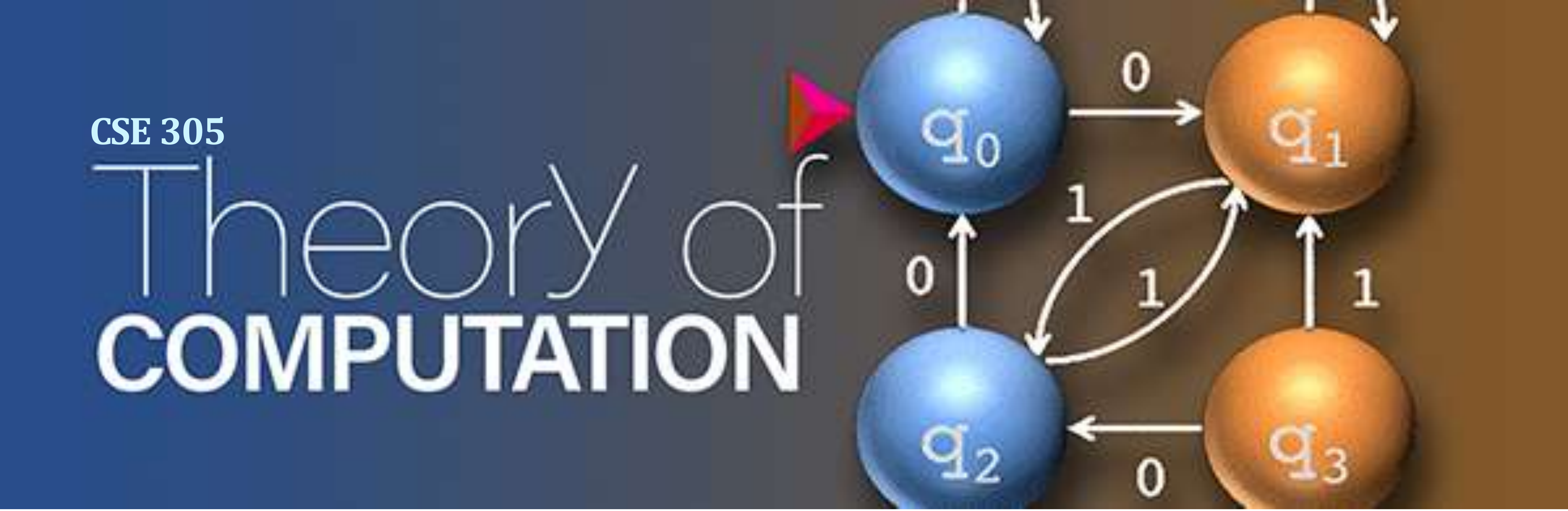

Lecture 18

Finite-State Automata (5)

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Contents

Finite State Automata

Equivalence of DFA and NFA

- It requires a method of converting NFA to its equivalent DFA.
- **In NFA, when a specific input is given to the current state, the machine goes to multiple states. It can have zero, one or more than one move on a given input symbol.**
- **On the other hand, in DFA, when a specific input is given to the current state, the machine goes to only one state. DFA has only one move on a given input symbol.**

- **The Equivalence of DFA's and NFA's:**
- Let, M = (Q, ∑, δ, q0, F) is an NFA which accepts the language L(M). There should be equivalent DFA denoted by M' = (Q', ∑', q0', δ', F') such that L(M) = L(M').

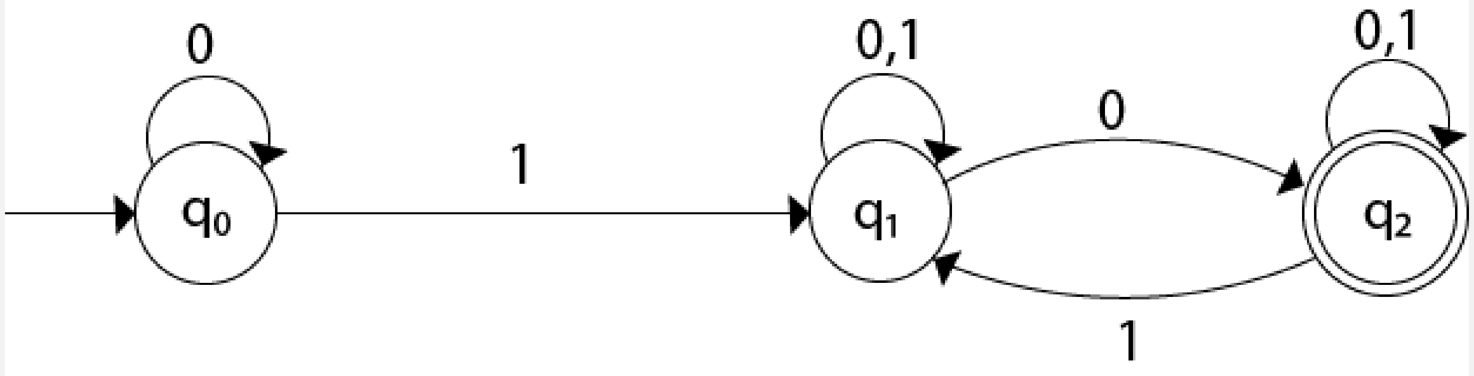
Conversion from NFA to DFA

Steps for converting NFA to DFA:

- **Step 1:** Initially Q' = ϕ
- **Step 2:** Add q0 of NFA to Q'. Then find the transitions from this start state.
- **Step 3:** In Q', find the possible set of states for each input symbol. If this set of states is not in Q', then add it to Q'.
- **Step 4:** In DFA, the final state will be all the states which contain F(final states of NFA)

Example: Conversion from NFA to DFA

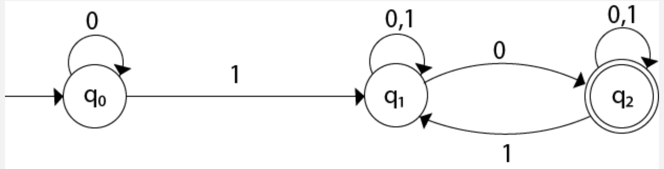
- **Example-1: Convert the following NFA to DFA.**



- **Solution:** For the given transition diagram we will first construct the transition table.

State	0	1
→q0	q0	q1
q1	{q1, q2}	q1
*q2	q2	{q1, q2}

Example: Conversion from NFA to DFA



- Now we will obtain δ' transition for state q0.

δ'([q0], 0) = [q0]

δ'([q0], 1) = [q1]

- The δ' transition for state q1 is obtained as:

δ'([q1], 0) = [q1, q2] (**new** state generated)

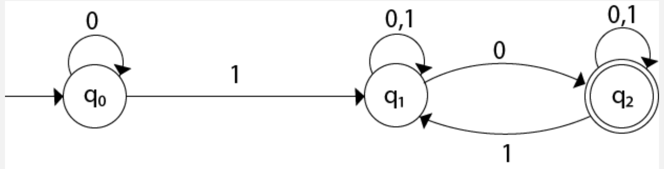
δ'([q1], 1) = [q1]

- The δ' transition for state q2 is obtained as:

δ'([q2], 0) = [q2]

δ'([q2], 1) = [q1, q2]

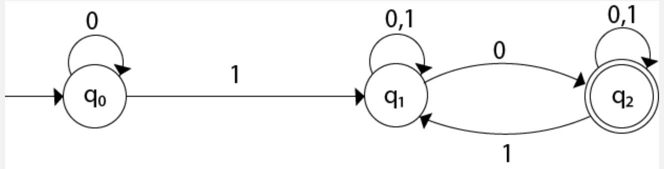
Example: Conversion from NFA to DFA



- **Now we will obtain δ' transition on [q1, q2].**

δ'([q1, q2], 0) = δ(q1, 0) ∪ δ(q2, 0)
= {q1, q2} ∪ {q2}
= [q1, q2]
δ'([q1, q2], 1) = δ(q1, 1) ∪ δ(q2, 1)
= {q1} ∪ {q1, q2}
= {q1, q2}
= [q1, q2]

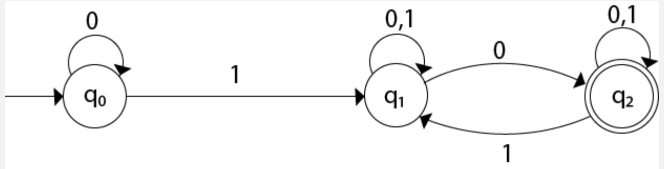
Example: Conversion from NFA to DFA



- **The state [q1, q2] is the final state as well because it contains a final state q2.**
- The transition table for the constructed DFA will be:

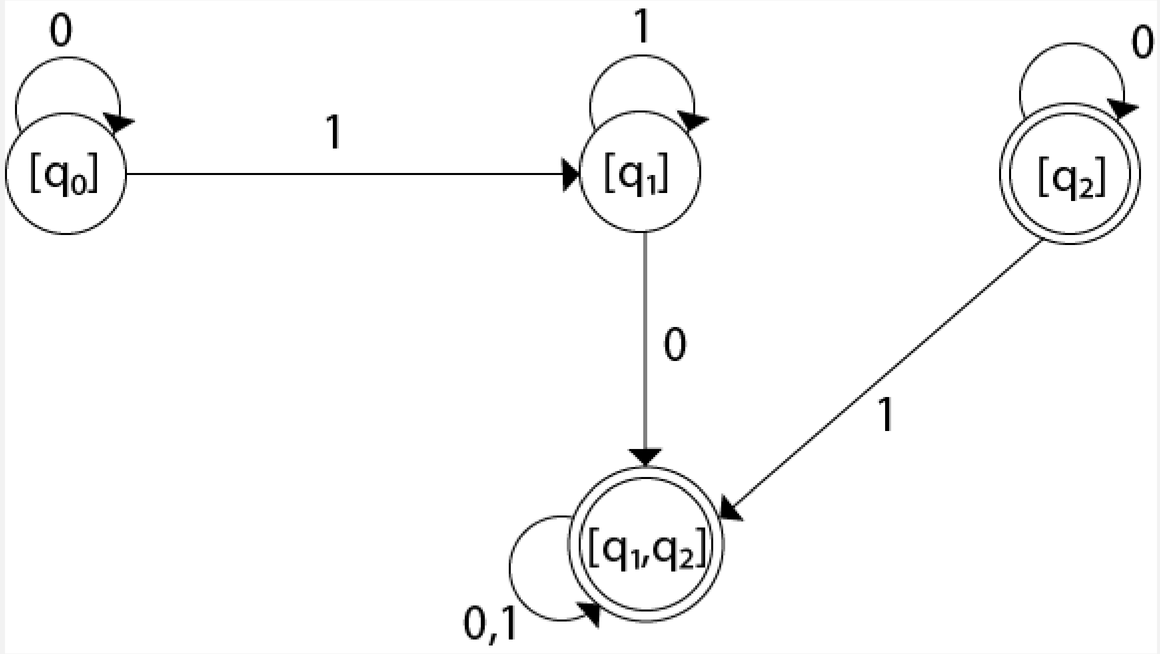
State	0	1
→[q0]	[q0]	[q1]
[q1]	[q1, q2]	[q1]
*[q2]	[q2]	[q1, q2]
*[q1, q2]	[q1, q2]	[q1, q2]

Example: Conversion from NFA to DFA



- **The state [q1, q2] is the final state as well because it contains a final state q2.**
- The Transition diagram will be:

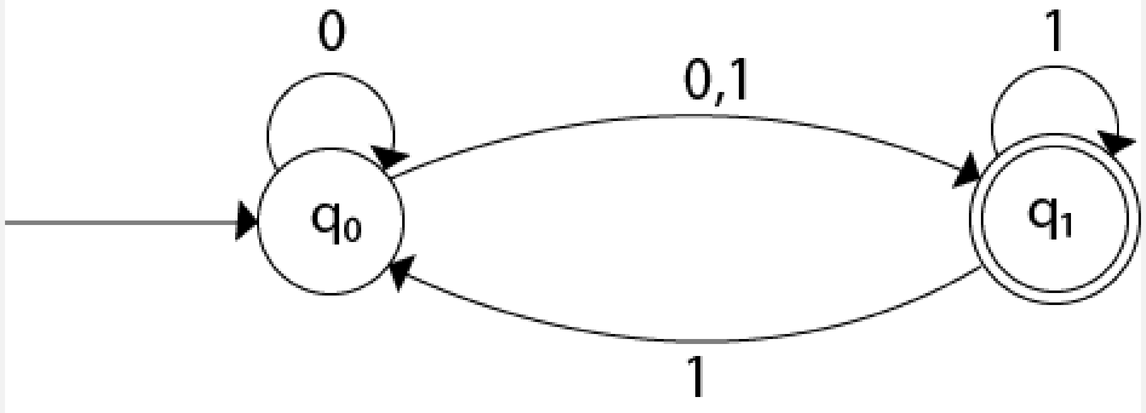
State	0	1
→[q0]	[q0]	[q1]
[q1]	[q1, q2]	[q1]
*[q2]	[q2]	[q1, q2]
*[q1, q2]	[q1, q2]	[q1, q2]



- **The state q2 can be eliminated because q2 is an unreachable state.**

Example: Conversion from NFA to DFA

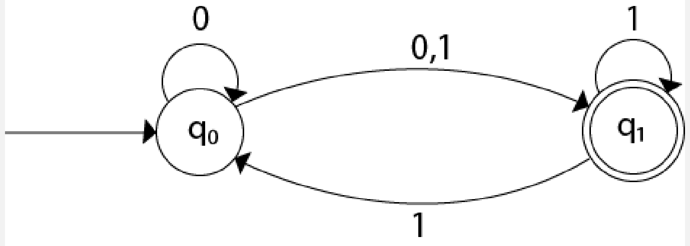
- **Example-2 : Convert the following NFA to DFA.**



- **Solution:** For the given transition diagram we will first construct the transition table.

State	0	1
→q0	{q0, q1}	{q1}
*q1	ϕ	{q0, q1}

Example: Conversion from NFA to DFA



- Now we will obtain δ' transition for state q0.

δ'([q0], 0) = {q0, q1}
= [q0, q1] (**new** state generated)
δ'([q0], 1) = {q1} = [q1]

- The δ' transition for state q1 is obtained as:

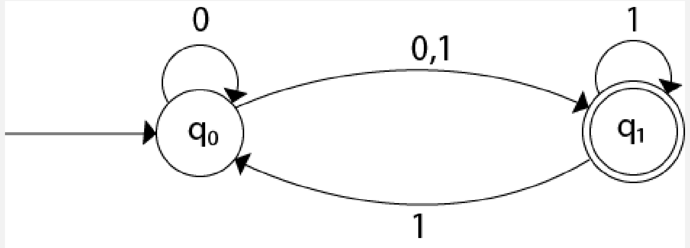
δ'([q1], 0) = ϕ
δ'([q1], 1) = [q0, q1]

- Now we will obtain δ' transition on [q0, q1].

δ'([q0, q1], 0) = δ(q0, 0) ∪ δ(q1, 0)
= {q0, q1} ∪ ϕ
= {q0, q1}
= [q0, q1]

δ'([q0, q1], 1) = δ(q0, 1) ∪ δ(q1, 1)
= {q1} ∪ {q0, q1}
= {q0, q1}
= [q0, q1]

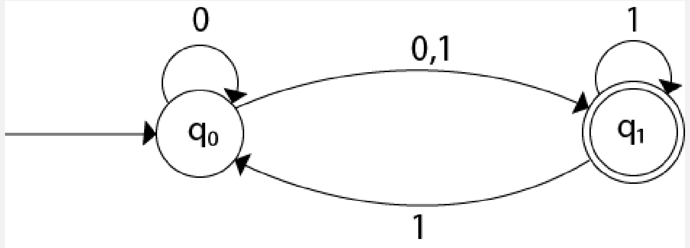
Example: Conversion from NFA to DFA



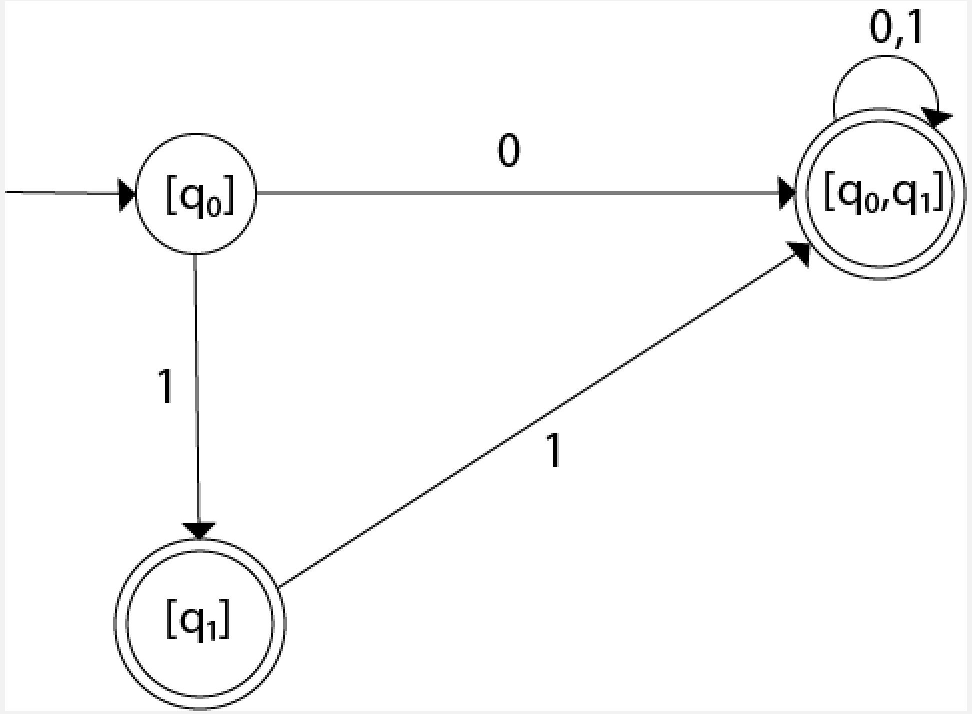
- As in the given NFA, q1 is a final state, then in DFA wherever, q1 exists that state becomes a final state. Hence in the DFA, final states are [q1] and [q0, q1].
- **Therefore set of final states F = {[q1], [q0, q1]}.**
- The transition table for the constructed DFA will be:

State	0	1
→[q0]	[q0, q1]	[q1]
*[q1]	ϕ	[q0, q1]
*[q0, q1]	[q0, q1]	[q0, q1]

Example: Conversion from NFA to DFA



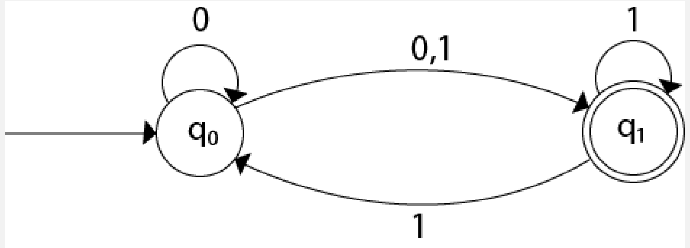
- **Therefore set of final states F = {[q1], [q0, q1]}.**
- The transition diagram for the constructed DFA will be:



State	0	1
→[q0]	[q0, q1]	[q1]
*[q1]	ϕ	[q0, q1]
*[q0, q1]	[q0, q1]	[q0, q1]

- Even we can change the name of the states of DFA.

Example: Conversion from NFA to DFA



NFA

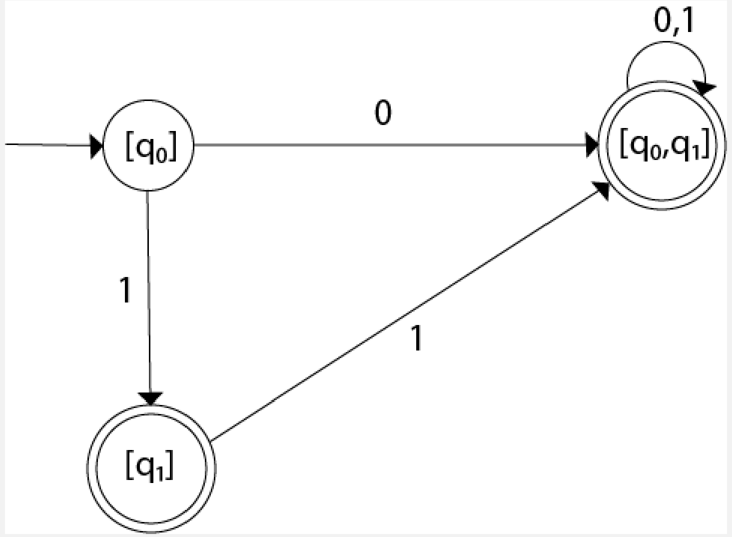
- Changing the name of the states of DFA.

Suppose:

A = [q0]

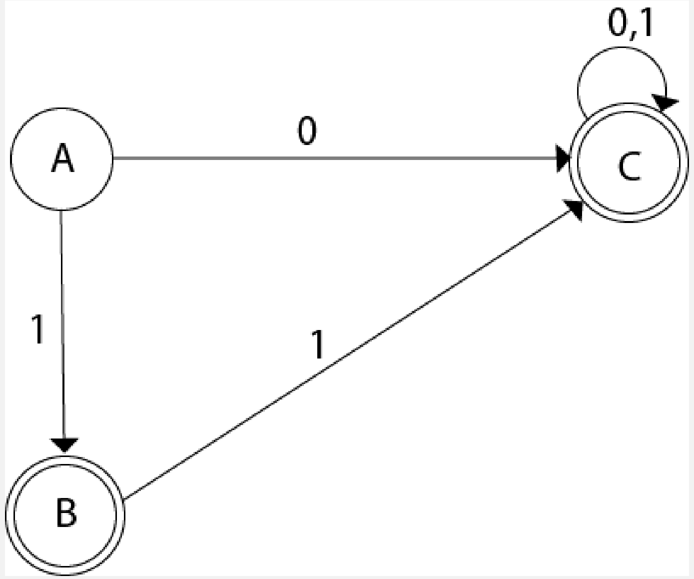
B = [q1]

C = [q0, q1]



DFA

- With these new names the DFA will be as follows:



Equivalence of DFA and NFA with e-Moves

- It requires a method of converting NFA with e-moves to its equivalent DFA.
- **Non-deterministic finite automata(NFA) is a finite automata where for some cases when a specific input is given to the current state, the machine goes to multiple states or more than 1 states. It can also contain ε move.** It can be represented as M = { Q, ∑, δ, q0, F}.

  Q: finite set of states

  ∑: finite set of the input symbol

  q0: initial state

  F: **final** state

  δ: Transition function

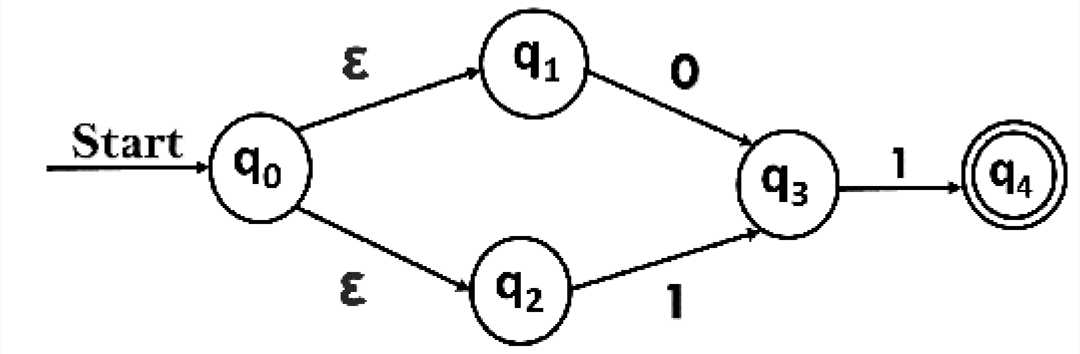
- **On the other hand, in DFA, when a specific input is given to the current state, the machine goes to only one state. DFA has only one move on a given input symbol.**
- **The Equivalence of DFA and NFA with e-moves:**
- Let, M = (Q, ∑, δ, q0, F) is an NFA with e-moves which accepts the language L(M). There should be equivalent DFA denoted by M' = (Q', ∑', q0', δ', F') such that L(M) = L(M’).

Conversion from NFA with ε to DFA

- **NFA with ∈ move:** If any FA contains ε transaction or move, the finite automata is called NFA with ∈ move.
- **ε-closure:** ε-closure for a given state A means a set of states which can be reached from the state A with only ε(null) move including the state A itself.
- **Steps for converting NFA with ε to DFA:**
  - **Step 1:** We will take the ε-closure for the starting state of NFA as a starting state of DFA.
  - **Step 2:** Find the states for each input symbol that can be traversed from the present. That means the union of transition value and their closures for each state of NFA present in the current state of DFA.
  - **Step 3:** If we found a new state, take it as current state and repeat step 2.
  - **Step 4:** Repeat Step 2 and Step 3 until there is no new state present in the transition table of DFA.
  - **Step 5:** Mark the states of DFA as a final state which contains the final state of NFA.

Example: Conversion from NFA with ε to DFA

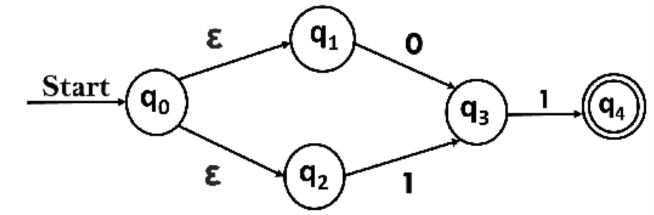
- **Example-1: Convert the following NFA with e-move into its equivalence DFA.**



- **Solution:** For the given transition diagram we will first construct the transition table.

States	Input e	Input 0	Input 1
q0	{q1, q2}	-	-
q1	-	q3	-
q2	-	-	q3
q3	-	-	q4
*q4	-	-	-

Example: Conversion from NFA with ε to DFA

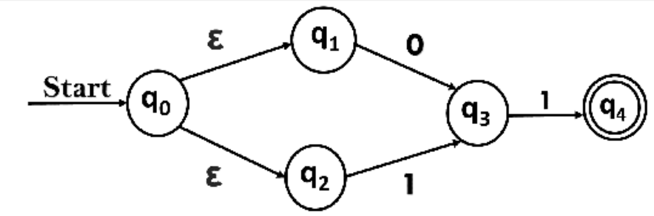


- Let us obtain ε-closure of each state.

> ε-closure {q0} = {q0, q1, q2}
>
> ε-closure {q1} = {q1}
>
> ε-closure {q2} = {q2}
>
> ε-closure {q3} = {q3}
>
> ε-closure {q4} = {q4}

- Now, let ε-closure {q0} = {q0, q1, q2} be state A.

Example: Conversion from NFA with ε to DFA

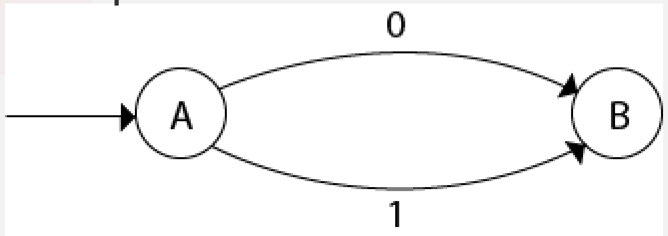


- Now, let ε-closure {q0} = {q0, q1, q2} be state A.
- **Hence,**

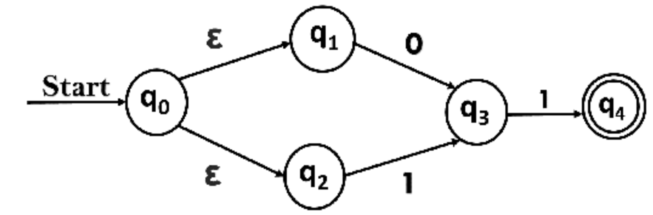
```
δ'(A, 0) = ε-closure {δ((q0, q1, q2), 0) }
         = ε-closure {δ(q0, 0) ∪ δ(q1, 0) ∪ δ(q2, 0) }
         = ε-closure {q3}
         = {q3}            call it as state B.

δ'(A, 1) = ε-closure {δ((q0, q1, q2), 1) }
         = ε-closure {δ((q0, 1) ∪ δ(q1, 1) ∪ δ(q2, 1) }
         = ε-closure {q3}
         = {q3} = B.
```

The partial DFA will be



Example: Conversion from NFA with ε to DFA



- **Now,**

```
δ'(B, 0) = ε-closure {δ(q3, 0) }
                = φ
δ'(B, 1) = ε-closure {δ(q3, 1) }
                = ε-closure {q4}
                = {q4}          i.e. state C
```

- **For state C:**

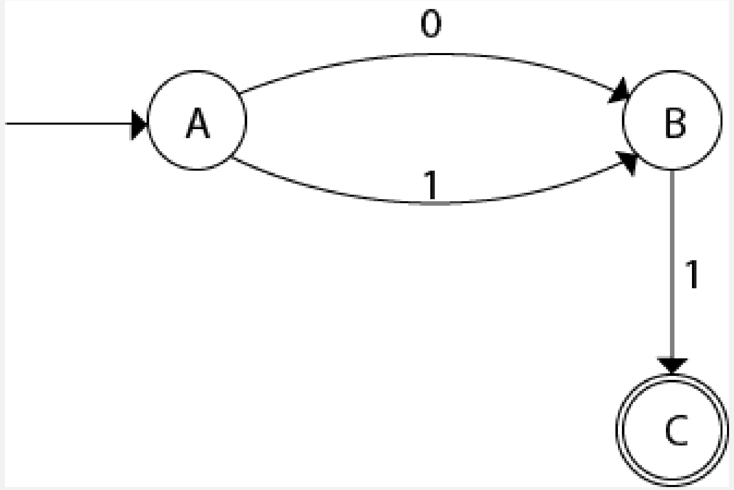
$\delta'(C, 0) = \varepsilon\text{-closure } \{\delta(q4, 0)\}$

$= \phi$

$\delta'(C, 1) = \varepsilon\text{-closure } \{\delta(q4, 1)\}$

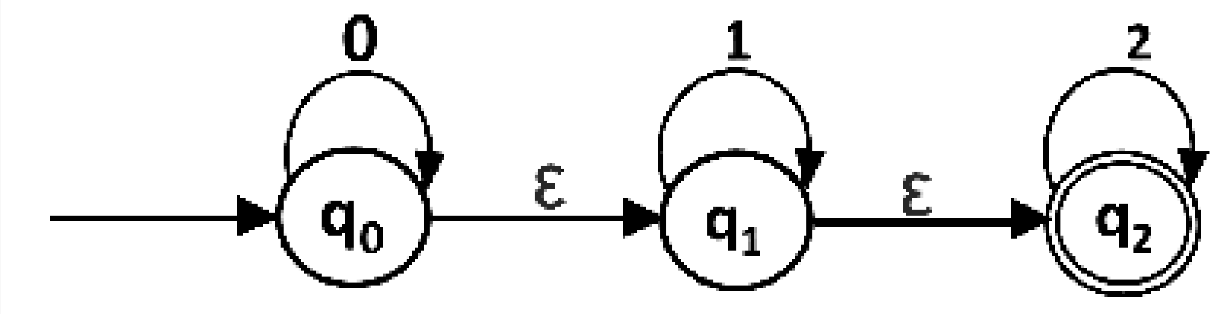
$= \phi$

The DFA will be:



Example: Conversion from NFA with ε to DFA

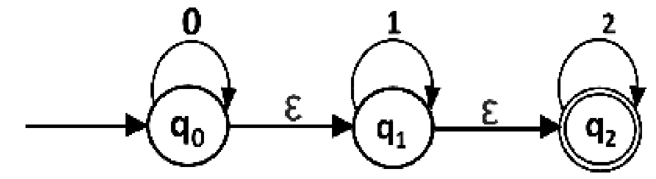
- **Example-2: Convert the following NFA with e-move into its equivalence DFA.**



- **Solution:** For the given transition diagram we will first construct the transition table.

States	Input e	Input 0	Input 1	Input 2
q0	q1	q0	-	-
q1	q2	-	q1	-
*q2	-	-	-	q2

Example: Conversion from NFA with ε to DFA



- Let us obtain the ε-closure of each state.

ε-closure(q0) = {q0, q1, q2}

ε-closure(q1) = {q1, q2}

ε-closure(q2) = {q2}

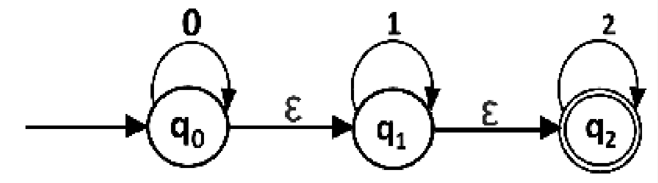
- Now we will obtain δ' transition. Let ε-closure(q0) = {q0, q1, q2} call it as **state A**.

```
δ'(A, 0) = ε-closure{δ((q0, q1, q2), 0)}
              = ε-closure{δ(q0, 0) ∪ δ(q1, 0) ∪ δ(q2, 0)}
              = ε-closure{q0}
              = {q0, q1, q2}

δ'(A, 1) = ε-closure{δ((q0, q1, q2), 1)}
              = ε-closure{δ(q0, 1) ∪ δ(q1, 1) ∪ δ(q2, 1)}
              = ε-closure{q1}
              = {q1, q2}            call it as state B
```

```
δ'(A, 2) = ε-closure{δ((q0, q1, q2), 2)}
              = ε-closure{δ(q0, 2) ∪ δ(q1, 2) ∪ δ(q2, 2)}
              = ε-closure{q2}
              = {q2}            call it state C
```

Example: Conversion from NFA with ε to DFA



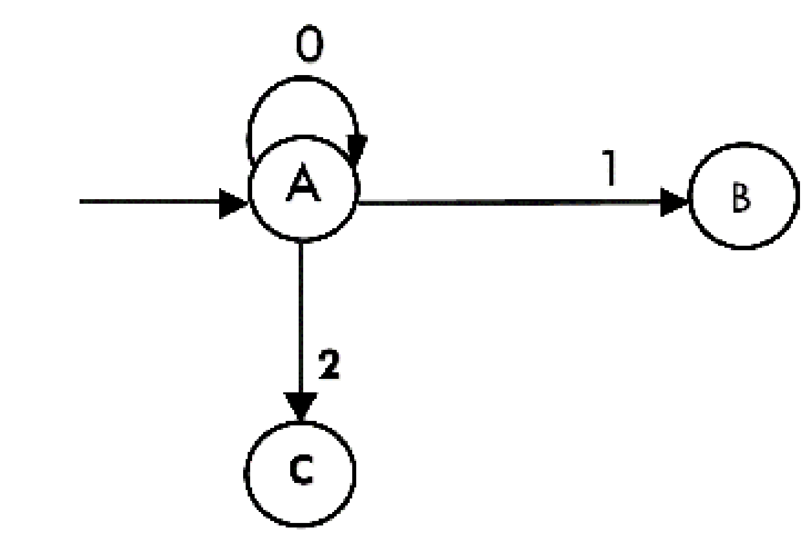
- Thus we have obtained

  $\delta'(A, 0) = A$

  $\delta'(A, 1) = B$

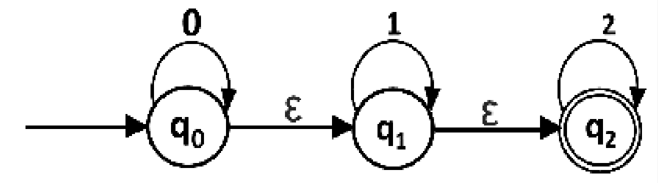
  $\delta'(A, 2) = C$

- The partial DFA will be:



- Now we will find the transitions on states B and C for each input.

Example: Conversion from NFA with ε to DFA

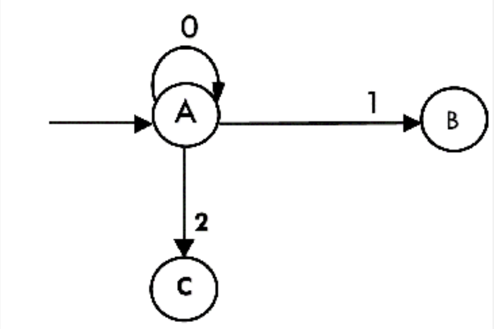


- Now we will find the transitions on states B and C for each input. Hence,

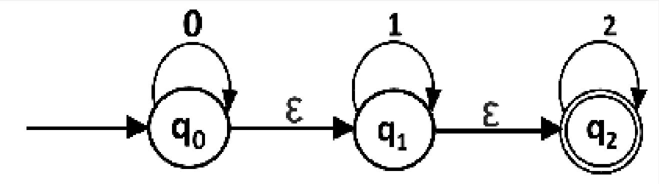
```
δ'(B, 0) = ε-closure{δ((q1, q2), 0)}
              = ε-closure{δ(q1, 0) ∪ δ(q2, 0)}
              = ε-closure{φ}
              = φ

δ'(B, 1) = ε-closure{δ((q1, q2), 1)}
              = ε-closure{δ(q1, 1) ∪ δ(q2, 1)}
              = ε-closure{q1}
              = {q1, q2}          i.e. state B itself

δ'(B, 2) = ε-closure{δ((q1, q2), 2)}
              = ε-closure{δ(q1, 2) ∪ δ(q2, 2)}
              = ε-closure{q2}
              = {q2}          i.e. state C itself
```



Example: Conversion from NFA with ε to DFA



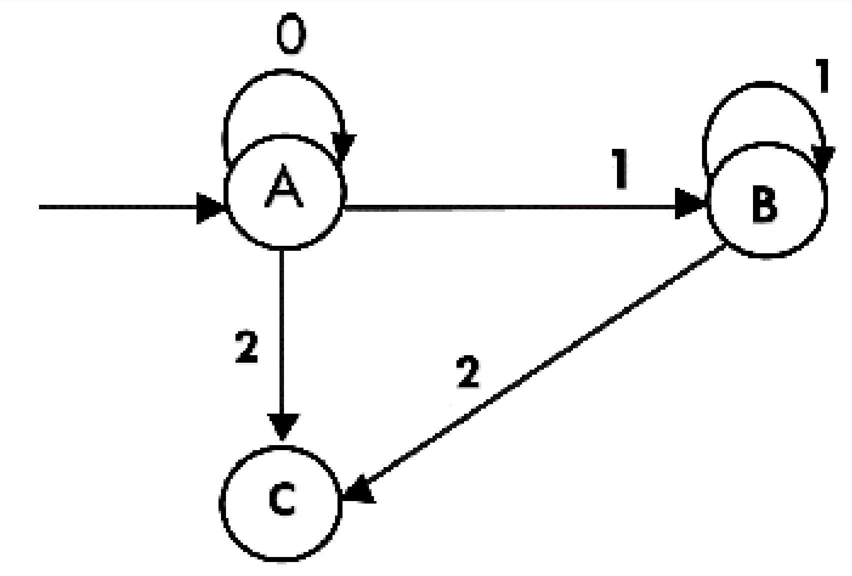
- Thus we have obtained

  $\delta'(B, 0) = \phi$

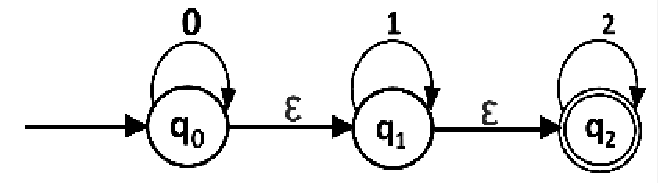
  $\delta'(B, 1) = B$

  $\delta'(B, 2) = C$

- The partial transition diagram will be



Example: Conversion from NFA with ε to DFA



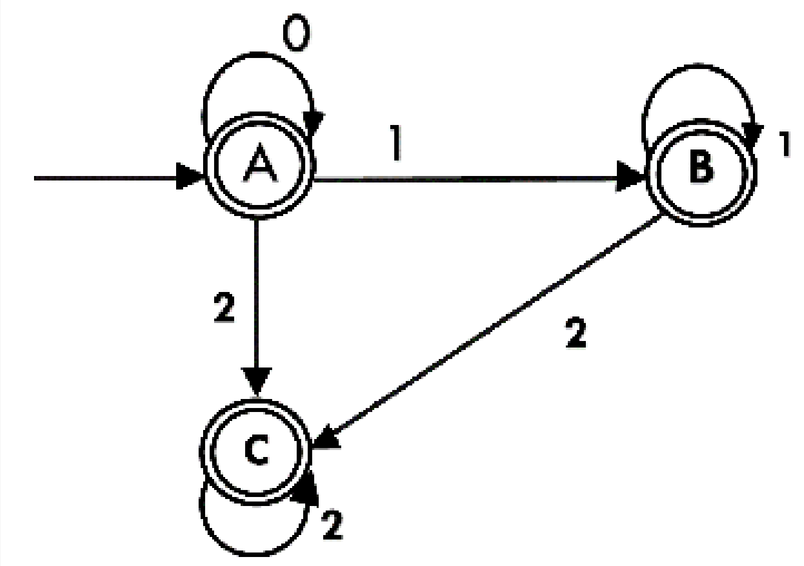
- Now we will obtain transitions for C:

```
δ'(C, 0) = ε-closure{δ(q2, 0)}
              = ε-closure{φ}
              = φ

δ'(C, 1) = ε-closure{δ(q2, 1)}
              = ε-closure{φ}
              = φ

δ'(C, 2) = ε-closure{δ(q2, 2)}
              = {q2}
```

Hence the DFA is:



- As A = {q0, q1, q2} in which final state q2 lies, hence A is final state. B = {q1, q2} in which the state q2 lies, hence B is also final state. C = {q2}, the state q2 lies, hence C is also a final state.

? THE END